A variety of trimming techniques for pseudo-Boolean proof logs

Arthur GONTIER et al

September 13, 2025

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Proof logs

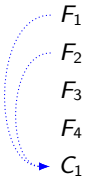
F_1

F_2

F_3

F_4

Proof logs



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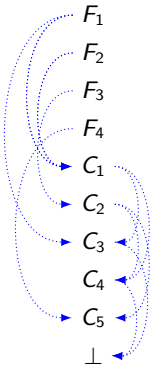

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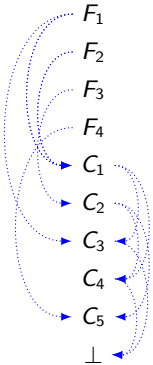

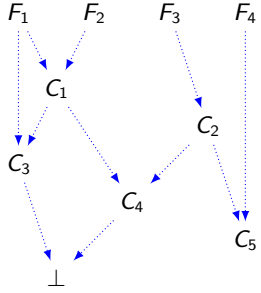

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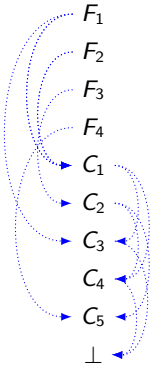

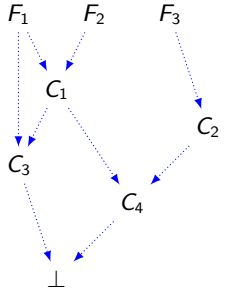

Looking for antecedents

pol F_1 3 * C_1 + s

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Antecedents: F_1, C_1

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pol F_1 3 * C_1 + s rup $ax + by + cz \geq r$

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red $ax + by \geq r$: x 0 : subproof
  proofgoal C_1
    pol C_2 F_3 + s
    rup $0 \geq 1$
  qed : -1
  proofgoal C_2
    pol C_1 F_2 + s
    rup $0 \geq 1$
  qed : -1
qed red

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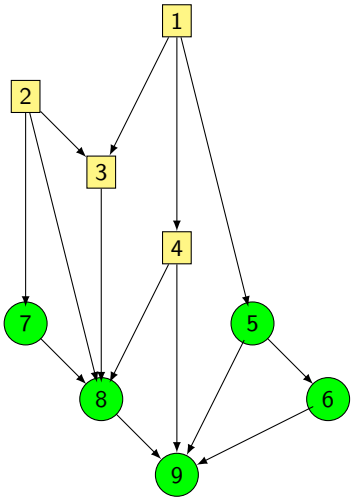
Antecedents: If proofgoal is marked then subproof antecedants are.

Precise deletion



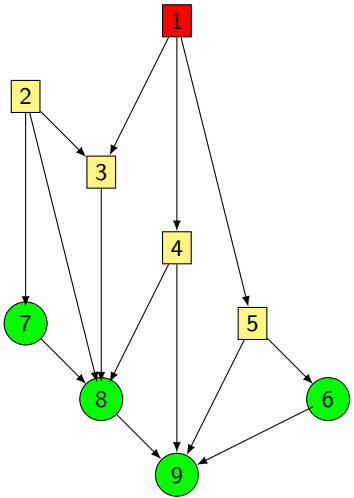
1

2

3

4

Precise deletion



1

2

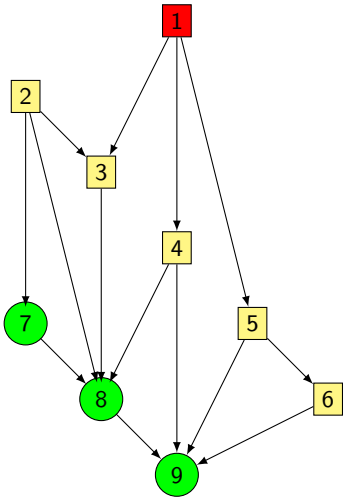
3

4

5

del id 1

Precise deletion



1

2

3

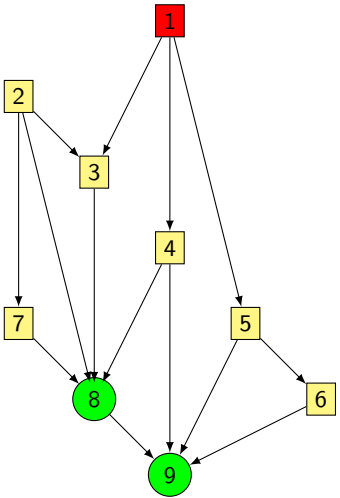
4

5

del id 1

6

Precise deletion



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2

3

4

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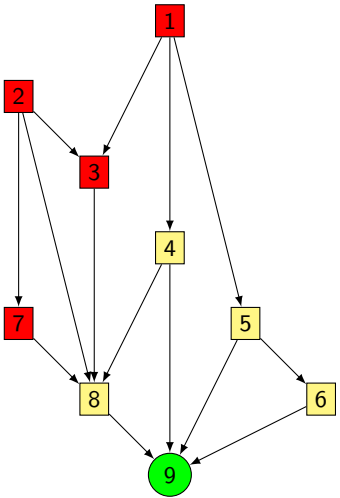
6

7

Precise deletion



1

2

3

4

5

del id 1

6

7

8

del id 2 3 7

Cone-first Rup

Algorithm 1: rup

```
1 while no contradiction do
2     propagate()
```

Cone-first Rup

Algorithm 2: rup

```
1 while no contradiction do
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```

Algorithm 3: Cone-first rup

```
1 only_marked ← true
2 while no contradiction do
3     if only_marked then
4         only_marked ← try_propagate_marked()
5     else
6         propagate_one()
7         only_marked ← true
```

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Trimmer for faster proof checks

1) Decoration :
   - Produce decorated proofs
   - Forward pass on the proof
   - Compute pol lines
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PPS: We can write to file each intermediate proof to avoid mem out on big proofs.

State of development (Berhan)

Implemented :

- Cone-first rup
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- Full-ram
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End goal :

- Supporting full veriPB grammar
- Code optimisation
- PB trimming paper

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Trimming for smaller proofs

Use cases:

- Using proof as certificate
- Try understanding your problem

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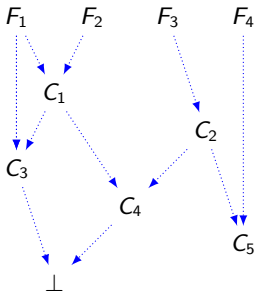
Philosophy:

- Take more time and resources to trim more
- Never trust Matthew



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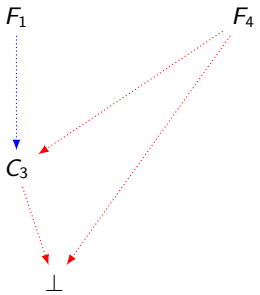

Do not trust Matthew's deletions

- A derived constraint may be useful later.

$C_{1.1}$
$C_{1.2}$
C_1
del id $C_{1.1}$ $C_{1.2}$
$C_{2.1}$
$C_{2.2}$
C_2
del id $C_{2.1}$ $C_{2.2}$
$C_{3.1}$
$C_{3.2}$
C_3
del id $C_{3.1}$ $C_{3.2}$
C_4
$\bot$

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C_2
C_3
del id $C_{1.1}$ $C_{1.2}$
C_4
$\bot$

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```
red ¬y ≥ 1 :  y 0
del id -1
red y ≥ 1 :  y 1
```

$C_{1.1}$
$C_{1.2}$
C_1
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C_2
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C_3
del id $C_{3.1}$ $C_{3.2}$
C_4
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```
red ¬y ≥ 1 :  y 0
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red y ≥ 1 :  y 1
```

- Fine if you don't reuse the names

$C_{1.1}$
$C_{1.2}$
C_1
del id $C_{1.1}$ $C_{1.2}$
$C_{2.1}$
$C_{2.2}$
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C_3
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Algorithm 4: rup

```
1 only_m ← true
2 while no contradiction do
3     if only_m then
4         only_m ← try_prop_m_ordered()
5     else
6         prop_one_ordered()
7         only_m ← true
8     update_implG(lit,lvl,ctrID)
9 MarkAntecedantsFromConflictAnalysis()
```

Better rup for trimming ?

- Cone-first
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Algorithm 5: rup

```
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6         prop_one_ordered()
7         only_m ← true
8     update_implG(lit,lvl,ctrID)
9 MarkAntecedantsFromConflictAnalysis()
```
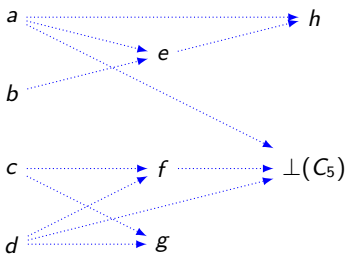

Algorithm 6: MAFCA

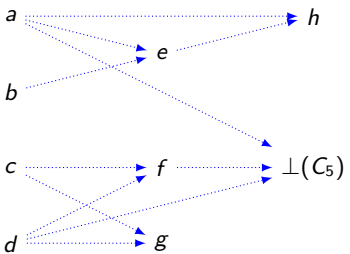
```
1  enqueue(PQ,⊥,1)
2  while not_empty(PQ) do
3      lit ← dequeue(PQ)
4      id ← implG[lit]
5      mark(id)
6      lits ← contributing_lits(id)
7      sort(lits,by:order)
8      for l ∈ lits do
9          if propagation then
10             break
11         else
12             set_lit(l,false)
13             enqueue(PQ,l,lvl)
```

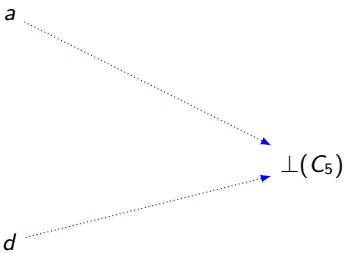
Conflict analysis example



$$\neg a + \neg d + \neg f \geq 2 \qquad (C_5)$$

Conflict analysis example





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- Idea: weaken unused literals in equations

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$$a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v+w+10x+y+z \geq 21$$

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$$a+b+c+d+e+10x \geq 1$$

Width trimming

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$$a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v+w+10x+y+z \geq 21$$

$$a + b + c + d + e + 10x \geq 1$$

- How to propagate this conelits through pol lines ?

Width trimming

- Idea: weaken unused literals in equations

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$$a + b + c + d + e + 10x \geq 1$$

- How to propagate this conelits through pol lines ?

$$a + \neg b + \neg c + d + e + 10x \geq 5 \quad (1)$$
$$2b + 2x \geq 1 \quad (2)$$
$$c + x \geq 1 \quad (3)$$
`pol 1 2 3 + +` (4)
`e` $a + b + d + e + 13x \geq 5$ (5)

Width trimming

- Idea: weaken unused literals in equations

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$$conelits \cup (poslits \cap neglits)$$

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- What about saturation and division ?

Visualisations

The graphs do not fit in the margin

Justification

1 Log proof skeleton

F_1

F_2

assert C_1

assert C_2

assert C_3

assert C_4

C_5

C_6

$\bot$

Justification

1. Log proof skeleton
2. Trim proof skeleton

F_1
F_2
assert C_1
assert C_2
assert C_3
assert C_4
C_5
C_6
$\bot$

F_1

assert C_2

assert C_4
C_5

$\bot$

Justification

1. Log proof skeleton
2. Trim proof skeleton
3. Justify to get a full proof
4. Normal trimming and/or checking

F_1
F_2
assert C_1
assert C_2
assert C_3
assert C_4
C_5
C_6
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F_1

assert C_2

assert C_4
C_5

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F_1
$C_{2.1}$
$C_{2.2}$
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- Visualisations
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End goal :

- Get proofs to be as small as possible
- Get satisfactory problem insight from small proofs
- PB trimming paper

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Some funny instances (These comparison are unfair)

	sizes			times (s)		
Instance	veriPB	fastPB	smallPB	veriPB	fastPB	smallPB (parse trim write verif)
LVg2g80	5.219 MB	6.455 MB	414.1 KB	44.2	12.5	100 (2.7 96.5 0.16 0.86)

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Instance	veriPB	fastPB	smallPB	veriPB	fastPB	smallPB (parse trim write verif)
LVg3g17	1.665 MB	499.4 KB	643.8 KB	0.46	0.79	1.19 (0.21 0.66 0.02 0.3)
bio027085	2.922 MB	333.2 KB	75.15 KB	4.28	32.4	3.7 (2.35 0.98 0.02 0.35)
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LVg3g41	8.664 MB	6.284 MB	421.1 KB	45.3	46.4	261 (4.72 245 1.02 9.94)

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LVg2g80	5.219 MB	6.455 MB	414.1 KB	44.2	12.5	100 (2.7 96.5 0.16 0.86)
LVg3g41	8.664 MB	6.284 MB	421.1 KB	45.3	46.4	261 (4.72 245 1.02 9.94)
bio170075	18.54 MB	14.15 MB	585.5 KB	192.0	173	571 (15.3 523 2.26 30.8)

Some funny instances (These comparison are unfair)

	sizes			times (s)		
Instance	veriPB	fastPB	smallPB	veriPB	fastPB	smallPB (parse trim write verif)
LVg3g17	1.665 MB	499.4 KB	643.8 KB	0.46	0.79	1.19 (0.21 0.66 0.02 0.3)
bio027085	2.922 MB	333.2 KB	75.15 KB	4.28	32.4	3.7 (2.35 0.98 0.02 0.35)
LVg2g80	5.219 MB	6.455 MB	414.1 KB	44.2	12.5	100 (2.7 96.5 0.16 0.86)
LVg3g41	8.664 MB	6.284 MB	421.1 KB	45.3	46.4	261 (4.72 245 1.02 9.94)
bio170075	18.54 MB	14.15 MB	585.5 KB	192.0	173	571 (15.3 523 2.26 30.8)
LVg53g56	46.81 MB	148.0 MB	12.89 MB	7.93	22.5	48.8 (10.2 32.9 1.95 3.84)

Conclusion

Merci pour votre attention !

Questions ?